Original article

Wake steering via yaw control in multi-turbine wind farms: Recommendations based on large-eddy simulation

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A B S T R A C T

The rotor area of a wind turbine must be perfectly perpendicular to the wind to deliver the most efficient performance as a single unit (i.e., yaw angle $\gamma = 0°$). When the rotor is misaligned (i.e., $\gamma \neq 0°$), the turbine power production is lower and greater structural loads may occur. However, a beneficial effect of yaw misalignment is that it steers the aerodynamic wake and, consequently, it may reduce wake losses for downstream wind turbines. To quantify and maximize such potential power gains while minimizing power losses, a wind farm with 28 wind turbines is investigated using large eddy simulations (LES). Selected turbines in the front-, mid-, and deep-rows are intentionally misaligned by positive (counter-clockwise) or negative (clockwise) yaw angles of various magnitudes. Consistent with previous studies, positive yaw misalignment angles cause net power gains, whereas negative ones cause net losses. We hypothesize that this is due to the Coriolis effect. Thus, only positive yaw misalignment angles should be considered for wake steering purposes in the northern hemisphere. Also, yawing front-row and deep-row turbines causes the overall power production to increase, while yawing mid-row turbines is not as effective. The most effective yaw misalignment angles are $+ 20°$ for front-row and $+ 10°$ for deep-row turbines.

Introduction

The large rotating blades of utility-scale horizontal-axis wind turbines extract energy from the undisturbed high-speed wind, leading to the creation of an expanding, low-speed, highly-turbulent region called the wake. The performance of a wind turbine located in the wake of upstream turbines is significantly lower than that of turbines receiving undisturbed wind. To put this into perspective, at Norrekaer, an onshore wind farm in Denmark with 13 Siemens 2.3 MW wind turbines positioned along a straight line, the turbulent wake produced by the front-row turbine causes a reduction in power production of the second-row turbine of approximately 60% for winds in the direction of alignment [1].

Therefore, wind turbines must be positioned within the given area of the farm in such a way that the overall negative impact of the upstream wakes is minimized (i.e., wind farm layout optimization). Numerous wind farm layout optimization algorithms have been developed over the last few years [2–11]. This tremendous progress impacts future wind farms, but it cannot improve the performance of the many existing wind farms that have already been developed and built. In order to improve the energy performance of existing wind farms, a wide range of potential techniques are being considered and evaluated to minimize the negative impact of upstream wakes by changing their direction in real time (i.e., wake steering). These techniques include yaw control [12–16], pitch control [17–21], tilt control [13,22], and torque control [19,23,24]. The present article studies the potential impact of yaw control on the power production of a utility-scale wind farm with 28 Siemens SWT-2.3–93 wind turbines. While the existing literature is mostly concerned with the impact of yaw misalignment angle on the wake characteristics of a single turbine, very few articles have investigated the effect of yaw misalignment on the performance of wind farms and most of them considered only a few wind turbines.

Bastankhah and Porté-Agel [12,25] studied the wake characteristics of a yaw-misaligned wind turbine via high-resolution particle image velocimetry measurements in a wind tunnel. Power and thrust measurements were also conducted to analyze the energy performance of the yaw-misaligned wind turbine. A simple and computationally inexpensive analytical model was developed to predict the wake deflection and the far-wake velocity distribution for yawed turbines. The deflection and morphology of wakes behind a single wind turbine operating in yawed conditions was also studied by Howland et al. [15] via wind tunnel experiments and large-eddy simulations (LES), where the wind turbine was modeled as a porous disk placed in a uniform inflow.

Fleming et al. [13] used LES of two inline wind turbines to study the

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impact of upstream yaw and tilt misalignment angles on wake deflection, power production, and dynamic loads. Several yaw misalignment angles ranging from $-35^\circ$ to $+40^\circ$ were tested. An important observation was that negative yaw misalignment angles caused a reduction in the total power production of the two wind turbines, while positive yaw misalignment angles increased it. The maximum power gain of 4.5% was observed at a yaw misalignment angle of $+25^\circ$. The impact of the upstream yaw angle on the performance of two inline wind turbines was also studied by Miao et al. [26], using the commercial computational fluid dynamics (CFD) software STAR-CCM+. Their results showed that the total power production of the unit of two inline wind turbines increased when the upstream wind turbine was intentionally yawed in the positive direction. In agreement with Fleming et al.’s [13] findings, Miao et al. [26–28] found: (i) negative upstream yaw misalignment angles cause a reduction in power production of the two inline wind turbines, (ii) the maximum power gain is 4.6%, which occurs when the yaw misalignment angle of the upstream wind turbine is $+25^\circ$.

Gebara et al. [16] conducted a yaw optimization for a wind farm consisting of six wind turbines (two parallel columns of three turbines each) with three different wind directions using a parametric model called FLOW Redirection and Induction in Steady-state (FLORIS). They found that yaw control is more effective for wind directions close to the direction of alignment. Using yaw control, the power production of the wind farm was increased by approximately 13% for wind directions within $5^\circ$ of the direction of alignment, while it was increased by only 1% for a wind direction that was misaligned by $-10^\circ$. For all three wind directions, the yaw optimization algorithm assigned relatively large yaw angles ranging from $19^\circ$ to $40^\circ$ to the front-row and the second-row wind turbines, while the optimal yaw angles of the third-row wind turbines were identified to be approximately zero (less than $1^\circ$). Fleming et al. [14] conducted a yaw optimization for the Princess Amalia Wind Farm, which is an offshore wind farm with 60 Vestas V80-2.0 MW wind turbines, using the FLORIS model and considering all wind directions with $5^\circ$ increments. They found that the yaw optimization increased the overall mean power output (and likewise power density) by 7.7%.

Wake steering via yaw control represents a paradigm shift in the way wind farms are operated, because the focus shifts from optimizing the individual turbine performance (old paradigm) to optimizing the entire wind farm collectively. However, misaligning the yaw of a wind turbine reduces its efficiency (because only the perpendicular component of the wind contributes to power generation) and may create excess loads on the turbine [29,30], thus it is a technique that should be used carefully. In addition, it is practically very challenging to control the yaw misalignment of all wind turbines of a farm in real time as the wind direction keeps changing and it is not uniform within a farm. The hypothesis assessed in this study is that, to boost power production of the wind farm, not all the wind turbines should be yawed and that there is an optimal subset of them that can be yawed without touching the rest. The main purpose of this investigation is to find general rules for yaw misalignment that can be used later for real-time optimization of wind farm controls.

### Methods

**Large eddy simulations (LES)**

LES govern the dynamics of large eddies by removing those with scales smaller than a filter length from the Navier-Stokes equations and modeling their effects using a subgrid-scale model. The filter length is defined as $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$, where $\Delta x$, $\Delta y$, and $\Delta z$ are cell sizes in the $x$, $y$, and $z$ directions, respectively. The incompressible formulations of the filtered continuity and momentum equations are as follows:

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad (1)
\]

\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = \frac{\partial \bar{p}}{\partial x_i} = \frac{\partial \bar{e}}{\partial x_i} - \frac{\partial \bar{\varepsilon}_f}{\partial x_i} + \frac{1}{\bar{\rho}_0} \frac{\partial \bar{p}_0 (x, y)}{\partial x_i} + F_{ext}, \quad (2)
\]

where the bar denotes spatially-resolved components; $u$ is the wind speed; $t$ is time; $\bar{p}$ is the modified pressure defined as:

\[
\bar{p} = \frac{p(x, y, z, t)}{\bar{p}_0} - \frac{p_0 (x, y)}{\bar{p}_0} + \frac{k}{3} + g z, \quad (3)
\]

$\bar{p}$ and $p_0$ are the mean and static pressure; $\bar{p}_0$ is the reference air pressure; $f$ is the modified state density; $\bar{\varepsilon}_f$ is the traceless part of the wind stress tensor; and $F_{ext}$ stands for the external forces applied to the wind, including those induced by the wind turbines. According to the Boussinesq eddy viscosity assumption, the traceless stress tensor $\bar{\varepsilon}_f$ given in Eq. (2) is defined as:

\[
\bar{\varepsilon}_f = -2 \nu_l \bar{S}_i, \quad (4)
\]

in which the kinematic eddy viscosity $\nu_l$ is defined using the subgrid scale model proposed by Smagorinsky [31] as:

\[
\nu_l = (c_D \Delta)^2 [S] \quad (5)
\]

where $c_D = 0.168$ is the Smagorinsky constant, $S = (\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i)/2$ is the filtered strain rate tensor, and $[S] = \frac{\sqrt{\Delta}}{\bar{\rho}_1}$ is the norm of the filtered strain rate tensor. The external force $F_{ext}$ term in Eq. (2) includes the Coriolis force, the buoyancy force, and the force exerted by turbine blades that is calculated using the actuator line model presented in the next section ‘The actuator line model’. Accordingly, the external force $F_{ext}$ can be expressed as:

\[
F_{ext} = -\frac{1}{\bar{\rho}_0} \bar{F}_l + \vec{f} \left\{ \frac{\partial \bar{\rho}}{\partial \bar{x}_i} - \frac{\partial \bar{\rho}}{\partial \bar{x}_k} \right\} \vec{u}_{ik} \vec{u}_{ik} \quad (6)
\]

where $\bar{F}_l$ is the force generated by the actuator line model, $\vec{f}$ is the alternating unit tensor, $g$ stands for the gravitational acceleration, $\vec{u}_i$ is the potential temperature, $\delta_\theta = 300K$ is the reference temperature, $\delta_\theta$ is the Kronecker delta, and $f$ is the Coriolis parameter defined as $f = 2 \Omega \sin \delta$, in which $\Omega$ is the Earth rotational speed ($\approx 2.95 \times 10^{-5}$ rad/s), and $\delta$ is the site latitude (here $\delta = 45^\circ$ N).

The following potential temperature equation needs to be solved coupled with Eqs. (1) and (2) to obtain the potential temperature needed to calculate the buoyancy term in Eq. (6):

\[
\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial (\bar{u}_i \bar{\theta})}{\partial x_i} = \frac{\partial \bar{q}_l}{\partial x_i} \quad (7)
\]

where $q_l$ represents the temperature flux defined as:

\[
q_l = -\frac{\bar{\rho} \vec{u}_i \vec{u}_i}{\bar{\rho} \Delta x} \quad (8)
\]

and $\bar{P}_\theta$ is the subgrid turbulent Prandtl number defined as [32]:

\[
\bar{P}_\theta = \frac{1}{1 + \frac{\partial \bar{\varepsilon}_f}{\partial \bar{\rho}}}, \quad (9)
\]

in which

\[
l = \begin{cases} \min (7.6 \bar{\varepsilon}_f^{0.5} (s^{1/2}) \Delta), & \text{if } s > 0 \\ \Delta, & \text{if } s \leq 0 \end{cases} \quad (10)
\]

\[
s = \frac{g \vec{u}_i \vec{u}_i}{\bar{\rho}} \Delta \quad (11)
\]

Usually $l = \Delta$, and hence, $\bar{P}_\theta = \frac{1}{7}$.

**The actuator line model**

The actuator line model, proposed by Sørensen and Shen [33], is usually employed along with LES to model the effect of wind turbines.
In this model, the turbine blades are represented by three rotating lines that are discretized into \( N_{\text{be}} \) blade elements \( (N_{\text{be}} = 21 \) in this study\) with centers located at \( (x_n, y_n, z_n) \). Following [34], we used airfoil lookup tables to replicate the aerodynamic properties of the proprietary Siemens SWT-2.3-93 (Table 1); the aerodynamic forces are calculated for each blade element \( f_{\text{xyzt}}(x_i, y_i, z_i, t) \). Summation of the aerodynamic forces of blade elements corrected via a regularization kernel yields the body force exerted by the blade onto the flow field, as follows:

\[
F_i = \sum_{n=1}^{N_{\text{be}}} f_{\text{xyzt}}(x_i, y_i, z_i, t) \pi r_{\text{n}}^2 \exp \left[-\frac{(r_{\text{n}})^2}{\varepsilon}\right].
\]

where \( f_{\text{xyzt}}(x_i, y_i, z_i, t) \) is the actuator element force, \( F_i \) is the force field projected as a body force onto the CFD grid, \( r_{\text{n}} \) is the distance between the CFD cell center and the blade element, and \( \varepsilon \) is used to control the Gaussian width so that it spans from the leading edge to the trailing edge of the blade elements. The value of \( \varepsilon \) is recommended to be \( l_c/4.3 \), where \( l_c \) indicates the chord length of the blade elements, so at both trailing and leading edges (i.e. \( r_{\text{n}} = l_c/2 \) the exponential term is reduced to approximately 1% of its maximum [35]. Aside from small frictional and other losses at the generator, multiplying the aerodynamic torque by the rotational speed of the rotor yields the power output.

**Numerical solution**

The governing equations described in the previous two sections ‘Large eddy simulations (LES)’ and ‘The actuator line model’ were solved using SOWFA (Simulator for Wind Farm Applications), which is a CFD solver based on the OpenFOAM toolbox developed by the United States National Renewable Energy Laboratory [34,36–38]. SOWFA was validated against observations at the Lillgrund wind farm (Sweden) and was found to perform well in past studies [34,39]. The equations were discretized using a finite volume formulation. A second order central differencing scheme was used to conduct the spatial discretization since the LES are too sensitive to false numerical diffusion and the central differencing scheme significantly reduces the numerical diffusion.

**Table 1**

Aerodynamic properties of the blade elements used in this study to replicate the Siemens SWT-2.3–93 wind turbine.

<table>
<thead>
<tr>
<th>Blade element</th>
<th>Radius (m)</th>
<th>Cord (m)</th>
<th>Twist (°)</th>
<th>Airfoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5714</td>
<td>2.036</td>
<td>9.000</td>
<td>Cylinder1</td>
</tr>
<tr>
<td>2</td>
<td>4.7143</td>
<td>2.065</td>
<td>9.000</td>
<td>Cylinder2</td>
</tr>
<tr>
<td>3</td>
<td>6.8571</td>
<td>2.334</td>
<td>9.000</td>
<td>Cylinder2</td>
</tr>
<tr>
<td>4</td>
<td>9.0000</td>
<td>2.736</td>
<td>9.000</td>
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</tr>
<tr>
<td>5</td>
<td>11.1429</td>
<td>3.137</td>
<td>9.000</td>
<td>FFA_W3-301</td>
</tr>
<tr>
<td>6</td>
<td>13.2857</td>
<td>3.485</td>
<td>9.000</td>
<td>FFA_W3-301</td>
</tr>
<tr>
<td>7</td>
<td>15.4286</td>
<td>3.372</td>
<td>9.000</td>
<td>FFA_W3-301</td>
</tr>
<tr>
<td>8</td>
<td>17.5714</td>
<td>3.183</td>
<td>9.000</td>
<td>FFA_W3-301</td>
</tr>
<tr>
<td>9</td>
<td>19.7143</td>
<td>2.995</td>
<td>8.079</td>
<td>FFA_W3-241</td>
</tr>
<tr>
<td>10</td>
<td>21.8571</td>
<td>2.807</td>
<td>6.014</td>
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</tr>
<tr>
<td>11</td>
<td>24.0000</td>
<td>2.618</td>
<td>4.231</td>
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</tr>
<tr>
<td>12</td>
<td>26.1429</td>
<td>2.430</td>
<td>2.589</td>
<td>FFA_W3-211</td>
</tr>
<tr>
<td>13</td>
<td>28.2857</td>
<td>2.242</td>
<td>1.303</td>
<td>FFA_W3-211</td>
</tr>
<tr>
<td>14</td>
<td>30.4286</td>
<td>2.054</td>
<td>0.281</td>
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</tr>
<tr>
<td>15</td>
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<td>1.865</td>
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<td>NACA_63-221</td>
</tr>
<tr>
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<td>-1.303</td>
<td>NACA_63-218</td>
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<tr>
<td>17</td>
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<tr>
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</tr>
<tr>
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<tr>
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<td>-2.842</td>
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<tr>
<td>21</td>
<td>45.4286</td>
<td>0.735</td>
<td>-2.973</td>
<td>NACA_63-218</td>
</tr>
</tbody>
</table>

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\]

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**Fig. 1.** Layout of the wind turbines in the four cases. Turbines are labeled with the column ID (A–H) and the row number (1–8, with the front-row at number 1). Yaw misalignment angles at the yawed turbines are shown in blue (negative) and red (positive). Column H in Case 3 is not used in this paper. The local refinement zone used for all cases is shown in grey in a).
comparison with the upwind scheme. The Pressure Implicit with Split Operator (PISO) [40] algorithm was used for the pressure-velocity coupling. Compared with other pressure-velocity coupling methods, such as SIMPLE [41] and SIMPLEC (SIMPLE-Consistent) [42], the PISO algorithm requires more CPU time per solver iteration; however, it significantly helps to maintain a stable calculation with larger time steps, and hence, decreases the number of iterations.

**Simulation setup**

The simulations were conducted as described in Vasel-Be-Hagh and Archer [7]. The computational area is a rectangular cuboid with width, depth, and height of 4000 m, 4000 m, and 1000 m, respectively. At first, a coarse mesh with approximately 5,900,000 hexahedral cells was created using the blockMesh utility supplied with OpenFOAM, corresponding to a grid resolution of 7 m in all directions. This mesh was then refined locally around the wind turbines to a grid resolution of 3.5 m (see grey-shaded area in Fig. 1a), leading to approximately 22,000,000 cells. The simulations were conducted in two steps. First, a “precursor simulation” was carried out to develop the turbulent flow field through the computational domain without taking into account the wind turbines for 12,000 s; then, a “wind plant simulation” (with turbines) was performed for 2000 s by adding the wind turbines into the developed flow field. Up to 28 Siemens SWT-2.3-93 wind turbines (with hub height \( H = 65 \) m and diameter \( D = 90 \) m) were placed in the domain, in various configurations described later, at streamwise and lateral distances of 4.3D and 6.6D respectively. Conducting the wind plant simulation over the above-described computational domain requires approximately 60,000 CPU-hours of 192 2.4-GHz processors of a high-performance computing cluster.

A mean wind speed of 9 m/s was specified at 90 m, the typical hub height of modern wind turbines (but not that of the Siemens SWT-2.3-93, which is 65 m), and from the 222° direction (measured clockwise from the north using the meteorological convention), coinciding with the direction of alignment of the turbine columns. The wind speed boundary condition was set as no-slip at the bottom boundary, slip at the top, outlet at the north and east boundaries, and inlet with time-varying mapped fixed values at the west and south boundaries. The surface roughness was set to 0.016 m, as in [39,7], and the turbulence intensity at the inlet was approximately 7%. The buoyant pressure boundary condition was used at all six boundaries of the domain to define the pressure boundary field. The temperature boundary condition was set as fixed gradient with a uniform value of 0.003 K m/s at the top boundary, zero gradient at the bottom, east and north boundaries (leading to neutral stability), and time-varying mapped fixed values at the south and west boundaries.

**Simulations**

Intentionally misaligning the yaw of a turbine causes a sharp drop in its power production and possibly adds undesirable stresses and loads on the rotor [29,30], although blade out-of-plane loads may actually be reduced [13]. As such, yaw misalignment should be done with caution and limited to a small subset of turbines, carefully selected to maximize the benefits. In this study, therefore, we tried to identify which rows in a wind farm are better candidates for yaw misalignment (the front, the middle, or even further downstream in a wind farm) and compared the benefits of varying the yaws of the turbines in those rows by positive vs. negative angles. Four wind farm simulations were conducted (Fig. 1), all with the same layout (with four columns of wind turbines, A-D, each with four to eight rows), but different combinations of yawed and non-yawed turbines. In each of the four simulations, one of the columns was not misaligned, to ensure a proper reference for comparison. Selected turbines in the other three columns were then misaligned by setting their yaw angles to specific non-zero values (between ±10° and ±30°), while retaining the same rotational speed as the non-yawed turbines. In the rest of the paper, mid-row and deep-row turbines are defined as those in the third and sixth rows, respectively. Each turbine will be identified by its column (A-D) and its row (1–8), e.g., turbine A6 is the sixth one in column A. The four cases were setup as follows:

1. **Case 1:** The first goal was to analyze the effect of imposing a yaw misalignment angle at the front-row turbines (A1 was misaligned by +20°, B1 by −10°, and D1 by +30°) on the downstream non-yawed turbines; the second goal was to assess the impact of misaligning the yaw of selected deep-row turbines (A6 by +20° and B6 by −30°). Column C was not misaligned (Fig. 1a) and was used as the reference for column C of cases 2, 3, and 4.
2. **Case 2:** We targeted the mid-row turbines (A3, B3, and C3) and the deep-row turbines (A6, B6, and C6) and misaligned the mid-row turbines by the same angle (+30°) while varying that of the deep-row turbines (A6: +30°; B6: +10°; D6: −30°). Column D was not misaligned (Fig. 1b) and was used as the reference for column D of cases 1, 3, and 4.
3. **Case 3:** We wanted to compare the effect of increasing vs. decreasing the positive yaw angle from the front- to the deep-row turbines (from +10° to +30° in A and from +30° to +10° in C). Column B was not misaligned (Fig. 1c) and was used as the reference for column B of cases 1, 2, and 4. Note that Column H does not coincide with Column D in the other cases, mistakenly, and therefore will not be analyzed in this paper.
4. **Case 4:** This case is similar in aim to Case 1 but for additional combinations of yaw misalignment angles (from +20° to −20° in B and from −20° to +20° in C). Column A was not misaligned (Fig. 1d) and was used as the reference for column A of cases 1, 2, and 3.

Unless otherwise stated, in the rest of the paper we will be comparing a yawed turbine against its corresponding turbine in the same column, but non-yawed, i.e., the reference column from the cases above. For example, to assess the effect of yaw misalignment of the front-row turbine by +30° on the third turbine, we will compare turbine C3 in Case 3 against turbine C3 in Case 1 and we will use the power of the reference, non-yawed, front-row turbine C1 from Case 1 to normalize the power of both columns.

**Wake detection and analysis**

Two properties of the wakes are of interest in this study, both of which are functions of downstream distance \( x \): (i) the position of the wake axis \( y(x) \) and (ii) the wake width \( w(x) \), both taken at a height of 80 m, approximately 15 m above hub height (65 m) to avoid the spurious effects caused by the lack of a nacelle. The wake center was assumed to be the barycenter of the momentum deficit [43]:

\[
\eta_m = \frac{\int y(1 - U^*)ds}{\int (1 - U^*)ds} \quad (13)
\]

\[
\zeta_m = \frac{\int z(1 - U^*)ds}{\int (1 - U^*)ds} \quad (14)
\]

where \( ds \) is a surface element over the area for which \( U^* < 1 \). \( U^* \) is the normalized stream-wise velocity (velocity in the x direction) defined as:

\[
U^* = \frac{u}{U_{in}} \quad (15)
\]

in which \( U_{in} \) is the undisturbed upstream wind speed. The integrals in Eqs. (13) and (14) are calculated over \( U^* < 1 \).

An alternative approach for locating the center of a wake is to find the location of the minimum wind speed (i.e., maximum speed deficit) [44,45]. Both approaches are used in the present work and the results are presented in the next section.
Results

The horizontal cross-sections of simulated hub-height mean wind speed in the four cases (Fig. 2) confirm that the wakes do not expand much laterally and therefore each column can be treated independently. In addition, since the incoming flow is turbulent, the mean wind speed is not horizontally homogeneous but presents streaks of higher and lower wind speeds. The same column in the four simulations is therefore characterized by the exact same upstream conditions, but these conditions vary from column to column. For example, the mean wind speed right upstream of Column A is lower than that right upstream of Column C, which means that the two front-row turbines A1 and C1 will be generating different amounts of power even when they are not misaligned, with C1 generating more than A1 (1272

Fig. 2. Simulated mean wind speed (m/s) at hub height (65 m) for the four cases in Fig. 1.

Fig. 3. Time-averaged wind speed (m/s) at hub height (65 m) with streamlines showing the wakes of the two yawed turbines.
vs. 1106 kW). This is why it was essential to have, for each column, a case in which the very same column was not yawed for reference.

Because there is no nacelle in the simulations, an artificial high-speed streak forms in all the wakes in correspondence of the missing nacelle (Fig. 2); however, it dissipates by $<2D$ downstream.

**Properties of the yawed wakes**

The deflection of a wake generated by a yawed turbine was investigated by analyzing the flow over two parallel, but independent, columns of wind turbines, B and C in Case 4 (see Fig. 3). Each column includes four turbines with the two front-row turbines misaligned by 20° in the opposite directions, i.e., one clockwise and one counterclockwise. Two streamlines passing through the two edges of each rotor were added to the two-dimensional velocity contour to provide a clearer illustration of the wake deflection caused by the yaw misalignment (Fig. 3). As observed in all previous studies, the wakes steer in the opposite direction of the rotor misalignment. When the rotor is yawed in the clockwise direction, the wake steers in the counterclockwise direction and vice versa.

The wake detection algorithms described in Section ‘Wake detection and analysis’ were used to capture the time-averaged wake trace downstream the yawed wind turbines (Fig. 4) in the near-wake region. By the time they reached the edge of the near-wake region, the wakes of the two yawed turbines were found to be deflected by 3° and 6° with respect to each other using the barycenter of the momentum deficit and the maximum deficit algorithms, respectively.

**Benefits of yaw misalignment at the front-row turbines**

We begin the discussion by looking at the benefits of misaligning the yaw angle of the front-row turbines. Intentionally misaligning the yaw of any turbine, including those in the front row, causes a sharp drop in their power production, proportional to the magnitude of the yaw misalignment angle. As shown in Fig. 5a, the relative power of the front-row turbines decreases by 3–27% for yaw misalignment angles between $\pm 10^\circ$ and $\pm 30^\circ$. The power loss of the front-row turbine is basically the same for positive and negative yaw misalignment angles of the same magnitude.

The general pattern for both positive (i.e., counter-clockwise) and negative (i.e., clockwise) yaw misalignment angles is a reduction in the front-row’s relative power and an increase in the next rows (Fig. 5a); furthermore, the front-row’s losses and the next rows’ gains in power are greater for larger magnitudes of the yaw misalignment angle (Fig. 5b). The second row behind the misaligned front-row has the most gain in power, although not as high as the loss in the front-row, and then the third row has a lower gain than the second row. The next turbines, from the fourth on, generally exhibit small gains in power when the front-row is yawed. However, these gains are of about the same order of magnitude as the standard deviation of power of non-yawed turbine and are actually losses at times (e.g., for $-30^\circ$ in

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**Fig. 4.** Time-averaged wake traces in the near-wake region downstream of the two yawed turbines presented in Fig. 3 using: a) the barycenter of the momentum deficit and b) the point with the maximum deficit. The wake traces were calculated at a height of 80 m, 15 m above hub height, to avoid the spurious acceleration caused by the lack of a nacelle.

**Fig. 5.** Effect of yaw misalignment angle of the front-row turbines on: a) relative power and b) power production (kW) of the downstream turbines. Relative power is calculated as the power of a turbine normalized by the power of the front-row turbine in the same column in the non-yawed reference case, as listed in Table 2. Similarly, the change in power production of a turbine is calculated with respect to the power of the same turbine in the non-yawed reference case. The warm-shade circles are for positive and the cool-shade triangles for negative yaw misalignment angles. The four non-yawed, reference cases are shown with black squares in a).
compared with term of Eq. (2) (also see Eq. (6)), this e

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Smaller (and gain in the third row by

explanations for this phenomenon in the literature. The

yawed wakes tend to de

Yaw misalignment angle Column Case Power loss Net power change

0° A 4 0 0

0° B 3 0 0

0° C 1 0 0

0° D 2 0 0

Front-row – 30° D 4 –272.0 –53.3

Front-row – 20° A 1 –145.8 –66.9

Front-row – 10° B 1 –29.5 –15.8

Front-row + 10° A 3 –41.8 +28.1

Front-row + 20° A 1 –142.2 +52.0

Front-row + 30° C 3 –337.5 +35.2

Front-row + 40° B 4 –134.1 +39.4

Front-row + 50° B 4 –134.1 +39.4

Front-row + 60° B 4 –134.1 +39.4

Mid-row + 30° A 2 –144.9 –60.1

Mid-row + 30° B 2 –145.2 –50.0

Mid-row + 30° C 2 –162.6 –8.7

Deep-row – 30° B 1 –108.3 –102.4

Deep-row – 30° C 2 –94.5 –86.5

Deep-row – 20° B 4 –18.8 –49.5

Deep-row – 10° B 2 +4.3 +50.7

Deep-row + 10° C 3 +0.9 +141.1

Deep-row + 20° A 1 –76.3 +16.8

Deep-row + 20° C 4 –55.5 +6.3

Deep-row + 30° A 2 –152.1 –32.2

Deep-row + 30° A 3 –144.4 –68.5

Table 2

Details of the yaw misalignment cases. The power loss is that at the first (or yawed) turbine in kW. The net power change is the sum of the changes in power of the first (or yawed) turbine plus the next two turbines downstream.

Thus we will focus our analysis hereafter only on the yawed turbine and the next two turbines downstream, for a total of three turbines in each case.

The sign of the yaw misalignment angle of the front-row matters, possibly more than its magnitude. Let us look at positive yaw misalignment angles first. For a front-row yaw misalignment angle of +20°, both cases (Column A in Case 1 and Column B in Case 4, orange circles in Fig. 5a) show a drop in relative power of the front-row turbine by 13%, followed by a gain in the second row by 11 – 13%, with another gain in the third row by 4 – 5%, causing an overall gain in the first three rows of 1 – 1.5% and a net power gain of 39.4–52.0 kW (Table 2). Smaller (+10°) and larger (+30°) positive yaw misalignment angles show the same pattern but less and more enhanced, respectively (Fig. 5b).

However, for negative yaw misalignment angles, the significant difference is that, consistent with previous studies, we too find that the net gain is negative. With the same loss in the front-row turbine as with a positive yaw misalignment angle, the turbines downstream of a negatively misaligned front-row turbine experience lower gains than with positively misaligned ones for the same magnitude of the yaw misalignment angle and not large enough to compensate for the loss in the front row. For example, Fig. 5 shows that the –30° curve (blue triangles) lies below the +30° curve (red circles), and so do the –20° and –10° compared with +20° and +10°.

The finding that negative yaw misalignment angles give a negative gain in power was also reported by other studies [13,26], but with no explanations. Here we propose that the reason why a positive (counter-clockwise) yaw misalignment angle gives higher benefits than a negative (clockwise) yaw misalignment angle resides in the fact that non-yawed wakes tend to deflect to the right, or clockwise. There are two explanations for this phenomenon in the literature. The first, discussed in [46], is that the Coriolis force in the northern hemisphere deflects a wind turbine wake to the right, or clockwise. Since the Coriolis force was included in $F_{Cor}$ term of Eq. (2) (also see Eq. (6)), this effect is included in the simulations performed for this study. An alternative explanation was proposed by [16], based on the results by [14], namely that the clockwise rotation of the turbine rotors (around the horizontal axis) causes the wakes to rotate counter-clockwise (around the horizontal axis), thus lower momentum to be advected upwards into the right side of the wake (looking downstream), which increases the wind speed deficit and causes the right shift of the wake. Since the turbines simulated here rotate clockwise, this mechanism also is included in our simulations.

Verifying the correctness of both or only one of these two theories is beyond the purpose of this study. What matters is the combined effect of this natural deflection to the right (clockwise) with the effects of the yaw misalignment. When a positive (counter-clockwise) yaw misalignment angle is imposed on a turbine, the wake is deflected clockwise, or to the right of the flow (see Section ‘Properties of the yawed wakes’). The wake is naturally deflected in exactly the same way (to the right or clockwise), therefore it enhances the clockwise deflection caused by the positive yaw misalignment. But with a negative (clockwise) yaw misalignment angle, the natural clockwise deflection acts against the counter-clockwise deflection caused by the yaw angle, thus the net result is that the wake is still deflected counter-clockwise, but less, ultimately not enough to reap the benefits.

To summarize the results obtained for the front-row yaw misalignment cases, Fig. 6 shows the changes in relative power (Fig. 6a) and power (Fig. 6b) as a function of the yaw misalignment angle, as well as the effect of averaging over the first three vs. four turbines. The dis-benefits of negative yaw misalignment angles are obvious in both figures. The question addressed in this figure is whether two or three non-yawed turbines behind the yaw-misaligned turbine are needed to obtain significant benefits in terms of power, thus three vs. four turbines in

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Fig. 6. Effect of the yaw misalignment angle of the front-row turbine on: a) the average relative power and b) the actual power generated by the first three or four turbines. The dotted lines represent the best polynomial fits.
Fig. 5 suggested that small gains occur even past the fourth turbine, but these gains appear to be of the same order of magnitude as the normal fluctuations of power (i.e., no-yaw cases) and therefore are not considered in Fig. 6. Whereas the relative power decreases substantially with four vs. three turbines (from around 61% to around 56%), a mere consequence of the fact that more turbines that produce less than the front-row turbine are being added to the mean, the actual changes in power are similar for three vs. four.

However, when only three are considered, the patterns of both relative power (Fig. 6a) and power change (Fig. 6b) are consistent, with a maximum around +20° or slightly less. In addition, it appears that considering three turbines may be a more conservative choice, as in most cases averaging over four turbines causes the net gain to increase with respect to the cases with averaging over three. As such, the discussion in this paper focuses on averages over three turbines, i.e., the yaw-misaligned one and the next two.

Looking at the results for the front-row in Table 2, it appears that a small, positive yaw misalignment angle (+10°) causes a small loss in power at the yawed turbines (−42 kW) and a relatively large net gain in power (+28 kW). By contrast, a large, positive yaw misalignment angle (+30°) may give a larger net gain (up to +35.2 kW), but with a much larger loss at the yawed turbine (−337 kW). Using an economic/financial analogy between dollars and kilowatts, in order to obtain a net gain of $35.2 with a yaw misalignment angle of +30°, one needs to be willing to invest upfront $337; for a yaw misalignment angle of +10°, one only needs to invest $42 in order to gain $28. An intermediate positive yaw misalignment angle (+20°), on the other hand, guarantees the largest net gains (over 39 kW) for a reasonable power loss at the front row (order of −130 kW). Thus we conclude that an intermediate, positive yaw misalignment angle is the optimal strategy for the front row.

Benefits of yaw misalignment at the mid- and deep-row turbines

Next, we look at the effects of yaw misalignment of the mid- and the deep-row turbines, defined as those at the third and sixth row, respectively (Fig. 7).

Focusing on the deep rows first, negative yaw misalignment angles in the deep rows (−20° and −30°) do not give any net benefit, but rather net losses (Fig. 7). Positive yaw misalignment angles in the deep rows may be beneficial, but not always. A large positive yaw misalignment angle (+30°) gives negative net power changes, whereas smaller positive yaw misalignment angles (+10° and +20°) give positive net power changes, although only with the smallest angle +10° the power gains are significant (order of 100 kW for +10° vs. 10 kW for +20° in Table 2). This is a noticeable difference with respect to misaligning the front row turbines, for which the +20° yaw misalignment angle was more effective than +10°. The reason for the large gain in power with a small positive yaw misalignment angle in the deep rows is that the yawed turbines do not really experience any power loss. As shown in Table 2, the two cases with a +10° yaw misalignment angle in the deep rows show either no loss or a small gain in power at the yawed turbines, while the next two non-yawed turbines still benefit from the small wake steering effect, ultimately giving the highest net gains (+68 and +141 kW) of all cases presented here. It remains unclear if this high power production by two deep row turbines despite the imposed +10° yaw misalignment angle is a coincidence or a consistent, general finding.

In the mid-rows (Fig. 7), there is no net power gain by misaligning the yaw angle by +30° in all three cases considered. Given that the power changes are similar to those in the deep-row cases discussed above (Table 2), but there is less turbulence and therefore less wind deflection in the mid-rows than in the deep-rows, it is unlikely that the positive benefits observed in the deep rows for +10° yaw misalignment angle be also found in the mid-rows. Future simulations with smaller positive yaw misalignment angles should be conducted to confirm this.

Benefits of positive yaw misalignment angles

Lastly, to summarize the effects of positive yaw misalignment angles only, we compare the effect of yawing front- versus mid- versus deep-row turbines on the power production of the yawed turbine and that of the next two turbines downstream (Fig. 8). For +30°, the benefits only manifest when the front-row turbine is yawed (although in one of the two cases the net benefit is actually slightly negative, but it becomes positive when the fourth turbine is included, see Fig. 6b); if a mid-row or a deep-row turbine is yawed by +30°, its drop in power is not compensated for by the next two turbines downstream and therefore the net result is a loss (Table 2). For +20°, there is a net gain in both front- and deep-row turbines, but the gain in the deep-row (+6.3 to +16.8 kW) is less than that in the front-row (+39.4 to +52.0 kW). For +10°, there is actually a larger gain in misaligning the yaw of the deep-row (+67.7 to +141.1 kW) than that of the front-row (+28.1 kW) turbine (Table 2).

Conclusions and applications

The advantages and disadvantages of wake steering via intentionally misaligning the yaw angles of selected turbines in a wind farm are investigated with the help of large eddy simulations (LES). Since yaw misalignment causes large power reductions and may increase stresses and loads on the yawed turbines, the approach proposed...
in this study is to limit the number of turbines that are misaligned. Rather than misaligning all turbines in a row, as done in previous studies, here only turbines in the front-, mid-, and deep-rows are misaligned with positive (counter-clockwise) or negative (clockwise) yaw misalignment angles between −30° and +30°. Four LES runs are conducted for four columns of Siemens SWT-2.3-93 turbines and the power output of each yawed column is compared with that of the same column but for a control, non-yawed reference case. Main findings include:

(1) As found in previous studies, the wake of a yawed turbine is narrower than the wake of the same non-yawed turbine and is shifted away from its axis by an angle that is opposite to the yaw misalignment angle. Thus the potential benefits of wake steering are due to both the deflection of the wake and the fact that the wake is narrower.

(2) The general pattern of power (and relative power) change in a column of turbines in which the first turbine is misaligned is that the first turbine experiences a drop in power that is proportional to the magnitude of the yaw misalignment angle (regardless of its sign), while the next non-yawed turbines experience a gain, when compared to the same column but with a non-yawed first turbine.

(3) The gains in power at the downstream non-yawed turbines do not compensate for the power losses at the first, misaligned turbine when the yaw misalignment angle is negative (clockwise). We propose that the explanation for this result is the Coriolis effect, which steers the wake naturally to the right (or clockwise), thus it counteracts the counter-clockwise turning of the wake induced by the yaw misalignment. Thus only positive yaw misalignment angles should be considered for wind farms in the northern hemisphere.

(4) For positive yaw misalignment angles, at least two turbines downstream of a yawed turbine are needed in order to get statistically significant net gains in power.

(5) Misaligning the yaw angle of front- or deep-row turbines is an effective strategy to increase the overall power production of a wind farm, while misaligning mid-row turbines is not as effective.

(6) Small, positive yaw misalignment angles (i.e., +10°) are a safe strategy for both front- and deep-row turbines, because the losses associated with yawing are small and the gains associated with the non-yawed turbines are small in absolute terms (kW), but relatively large when compared to the losses themselves (ratio of net gain over loss >67%).

(7) For the front-row, an intermediate positive yaw misalignment angle (i.e., +20°) is the most effective strategy, giving high net gains in kW for a moderate loss in power at the yawed turbines (ratio of net gain over loss >37%). Large, positive yaw misalignment angles (i.e., +30°) are not recommended because the ratio of net gain over loss is small (<10%).

(8) For the deep-rows, the recommended strategy is to apply a small, positive misalignment angle (i.e., +10°) because the power losses caused by it at the yawed turbines do not actually occur, thus the net gains are the highest of all cases analyzed here (net gains exceed the losses by at least a factor of 15). Further studies are recommended to verify if this finding is specific to the configurations studied here or a general result. Large, positive yaw misalignment angles (i.e., +30°) should not be used because they cause no gains but net power losses.

Whereas for new wind farms many optimization opportunities are available, from the layout of the turbines, to their model, blade length, or hub height, to their controls (including yaw), for existing wind farms most of these opportunities do not exist and wake steering via yaw control is one of the few that are available. The findings in this study are useful to wind turbine manufacturers, who could become more competitive by offering local as well as global optimization systems, but also to those wind farm operators and owners who want to develop innovative strategies to maximize annual power production using yaw control, but do not want to risk shortening the life span of their turbines by using yaw misalignment excessively.

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