Wind farm hub height optimization

Ahmadreza Vasel-Be-Hagh*, Cristina L. Archer

Physical Ocean Science and Engineering, University of Delaware, Newark, DE 19716, USA

HIGHLIGHTS

- Wind farms with multiple hub height turbines produce >2% more energy annually.
- Maximum benefits of multiple hub heights are found offshore with tight spacing.
- Shear loss and unexposed area gain are competing effects of multiple hub heights.
- Large eddy simulation confirmed PARK model's benefits of multiple hub heights.
- PARK model underestimates power gains of multiple hub heights by a factor of 2.

ARTICLE INFO

Article history:
Received 8 December 2016
Received in revised form 28 February 2017
Accepted 21 March 2017

Keywords:
Wind farm
Hub height
Wake losses
Optimization
PARK model
Large eddy simulation

ABSTRACT

The objective of this study is to assess the effect of hub height optimization on the Annual Energy Production (AEP) of a wind farm. The only optimization variable is the hub height of each wind turbine and all other characteristics of the wind farm, including base location, rotor diameter (D), and total number of wind turbines, remain unchanged. The first case study consisted of two wind turbines aligned with the wind direction, with the hub height of the upstream turbine fixed and that of the downstream turbine varying. Two competing effects were identified. On one hand, lowering the hub height of the downwind turbine exposes a larger fraction of the downwind rotor to the undisturbed wind and therefore increases its power production. On the other hand, turbines with lower hub heights experience lower wind speeds due to shear. The balance between these two effects is complicated by surface roughness and spacing between turbines. The maximum benefits are found offshore, with tight axial spacing (6D). To put into perspective the effectiveness of using multiple hub heights in a realistic setting, the second case compared two 20-turbine wind farms with the same horizontal layout, turbine type, and wind direction, but one with all turbines installed at the same hub height (80 m) and the second with alternating rows of tall (100 m) and short (57.5 m) wind turbines. The vertically staggered configuration was found to produce approximately 5.4% more power. Lastly, a real 48-turbine wind farm (Lillgrund in Sweden) was optimized under all 360 wind directions and wind speeds ranging from the cut-in to the cut-out using a greedy search algorithm. The optimal configuration presented a non-intuitive and balanced mix of 26 tall and 22 short turbines. Although the entire range from a minimum to a maximum predefined hub height was searched to determine the optimal hub height for each wind turbine, the optimization algorithm ended up with an optimal layout in which the hub height of each turbine was equal to either the minimum hub height or the maximum hub height and no value in between was assigned to any of the turbines. The AEP of the optimized wind farm with multiple hub heights was approximately 2% (10 GWh) higher than that of the wind farm with a single hub height. The PARK model was used in this study to calculate wake losses and to perform the optimization. To validate its results, Large-Eddy Simulations (LES) of the flow around the turbines, represented as actuator lines, were conducted at fine spatial and temporal resolutions. The LES results confirmed that optimizing hub heights is beneficial and indicated that the net gains were actually higher (by up to a factor of two) than those calculated with PARK.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

The renewable energy industry in general, and wind energy in particular, has expanded significantly in recent years. The total installed wind capacity in the United States experienced an
approximately 30-fold increase from 2000 to 2015, with installed wind capacity reaching 73,992 MW [1]. Despite this significant expansion, major efforts are still in progress to improve the efficiency of wind power conversion technology by optimizing the aerodynamic performance of wind turbines [2,3], the structural design of wind turbines [4,5], control strategies [6–8], the site selection [9,10], and the layout of wind farms [11–18]. Our review on the latter area, i.e., wind farm layout optimization, indicated that the focus of the investigations has been mostly limited to the latter area, i.e., wind farm layout optimization, indicated that the focus of the investigations has been mostly limited to the objectives of that study was to assess the effectiveness of the above-mentioned wake models in predicting power production of a wind farm with multiple hub heights, not to optimize the hub height of wind turbines, and hence, the base location and hub height of each wind turbine was assigned in advance. There are other studies that consider wind farms with multiple hub heights, however, they do not take into account the hub height as an optimization parameter, for instance, the wind tunnel study conducted by Chamorro et al. [33] and the large eddy simulations of a hybrid vertically staggered wind farm conducted by Xie et al. [34].

In a recent investigation, Wang et al. [29] studied a wind farm that includes 30 wind turbines with 3 different hub heights of 40, 60, and 80 m using PARK, Larsen, and BPA (Bastankhah-Porté Agel) wake models and compared the results of each wake model with those of a RANS turbulence model produced by the ANSIYS- Fluent commercial CFD package in 3 specific wind directions. The objective of that study was to assess the effectiveness of the above-mentioned wake models in predicting power production of a wind farm with multiple hub heights, not to optimize the hub height of wind turbines, and hence, the base location and hub height of each wind turbine was assigned in advance. There are other studies that consider wind farms with multiple hub heights, however, they do not take into account the hub height as an optimization parameter, for instance, the wind tunnel study conducted by Chamorro et al. [33] and the large eddy simulations of a hybrid vertically staggered wind farm conducted by Xie et al. [34].

DuPont et al. [30] employed an Extended Pattern Search (EPS) - Multi Agent System (MAS) algorithm along with the PARK wake model to design a wind farm over a 2000 m \( \times 2000 \) m area by optimizing number, base location, diameter, and hub height of the wind turbines, with a cost objective function. Two wind scenarios were considered; first, a constant wind direction (south) with a constant wind speed of 10 m/s, and, second, 36 wind directions with three wind speeds of 6, 9, and 12 m/s under stable, unstable, and neutral atmospheric conditions. In addition to the number of

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>rotor area [m(^2)]</td>
</tr>
<tr>
<td>( C_P )</td>
<td>power coefficient</td>
</tr>
<tr>
<td>( C_T )</td>
<td>thrust coefficient</td>
</tr>
<tr>
<td>( c_w )</td>
<td>scale factor</td>
</tr>
<tr>
<td>( D )</td>
<td>rotor diameter [m]</td>
</tr>
<tr>
<td>( d )</td>
<td>axial spacing between turbines [m]</td>
</tr>
<tr>
<td>( f )</td>
<td>coriolis parameter [rad s(^{-1})]</td>
</tr>
<tr>
<td>( f^a )</td>
<td>actuator element force [N]</td>
</tr>
<tr>
<td>( f_r )</td>
<td>frequency</td>
</tr>
<tr>
<td>( H )</td>
<td>hub height [m]</td>
</tr>
<tr>
<td>( h )</td>
<td>height [m]</td>
</tr>
<tr>
<td>( k )</td>
<td>decay coefficient</td>
</tr>
<tr>
<td>( k_w )</td>
<td>shape factor</td>
</tr>
<tr>
<td>( l )</td>
<td>mixing length [m]</td>
</tr>
<tr>
<td>( l_c )</td>
<td>chord length [m]</td>
</tr>
<tr>
<td>( P )</td>
<td>power [kW]</td>
</tr>
<tr>
<td>( P_c )</td>
<td>pressure [Pa]</td>
</tr>
<tr>
<td>( P_{\text{rated}} )</td>
<td>rated power [kW]</td>
</tr>
<tr>
<td>( P_w )</td>
<td>Weibull probability density</td>
</tr>
<tr>
<td>( q_\tau )</td>
<td>temperature flux</td>
</tr>
<tr>
<td>( r )</td>
<td>distance between cell center and blade element [m]</td>
</tr>
<tr>
<td>( S )</td>
<td>strain rate tensor [s(^{-1})]</td>
</tr>
<tr>
<td>( t )</td>
<td>time [s]</td>
</tr>
<tr>
<td>( u )</td>
<td>wind speed [m s(^{-1})]</td>
</tr>
<tr>
<td>( u_{\text{rated}} )</td>
<td>rated wind speed [m s(^{-1})]</td>
</tr>
<tr>
<td>( x )</td>
<td>( x ) axis in Cartesian coordinates</td>
</tr>
<tr>
<td>( y )</td>
<td>( y ) axis in Cartesian coordinates</td>
</tr>
<tr>
<td>( z )</td>
<td>( z ) axis in Cartesian coordinates</td>
</tr>
<tr>
<td>( z_0 )</td>
<td>surface roughness [m]</td>
</tr>
</tbody>
</table>

### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>induction factor</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>filter width [m]</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Kronecker delta, wind speed deficit [m s(^{-1})]</td>
</tr>
<tr>
<td>( \delta_H )</td>
<td>hub height increment [m]</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>alternating unit tensor</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>projection width [m]</td>
</tr>
<tr>
<td>( \theta )</td>
<td>temperature [K]</td>
</tr>
<tr>
<td>( \nu )</td>
<td>viscosity [m(^2) s(^{-1})]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density [kg m(^{-3})]</td>
</tr>
</tbody>
</table>

### Acronyms

- **AEP** : Annual Energy Production
- **BPA** : Bastankhah-Potré Agel
- **CFD** : Computational Fluid Dynamics
- **EPS** : Extended Pattern Search
- **LES** : Large Eddy Simulation
- **MAS** : Multi Agent System
- **PISO** : Pressure Implicit with Split Operator
- **Pr** : Prandtl number
- **SIMPLE** : Semi-Implicit Method for Pressure-Linked Equations
- **SOWFA** : Simulator fOr Wind Farm Applications
- **SSR** : Sub-Synchronous Resonance
- **RANS** : Reynolds Averaged Navier-Stokes

### Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_t )</td>
<td>0.168 Smagorinsky constant</td>
</tr>
<tr>
<td>( g )</td>
<td>9.81 gravitational acceleration [m s(^{-2})]</td>
</tr>
<tr>
<td>( n_h )</td>
<td>8760 number of hours in one year</td>
</tr>
</tbody>
</table>
turbines and the base location of each turbine, a range of hub heights and a range of rotor diameters were assigned to each turbine to minimize costs and maximize power production of the wind farm. For instance, for the 36 wind direction case where the vertical wind shear profile and the wake decay constant were set to represent neutral atmospheric condition, the optimized wind farm was found to have 21 wind turbines where 8 of them should have a rotor diameter in the range of 80–108 m, 7 of them should have a rotor diameter in the range of 80–100 m and 6 of them should have a rotor diameter in the range of 60–78 m and a hub height in the range of 60–79 m, and the rest of the turbines should have a rotor diameter and a hub height both larger than 120 m.

Chen et al. [31] used a greedy algorithm along with PARK to minimize cost per unit power output of a wind farm with 22 wind turbines where the hub height of each wind turbine could be 50 or 78 m. Two wind scenarios were considered; first, a constant wind direction (south) with three constant wind speeds of 12, 13 or 14 m/s, and, second, 16 wind directions with wind speed magnitude ranging between 10 and 14 m/s. At the constant wind direction, the performance of the wind farm including wind turbines with both hub heights of 50 m and 78 m was slightly better than the performance of the wind farm with a constant hub height of 78 m for all 12, 13, and 14 m/s wind speeds. It is worth mentioning that the base location of the wind turbines for those two wind farms were different, hence, the difference in the objective function can not only be attributed to using multiple hub heights. With the 16-direction scenario, however, the performance of the wind farm including wind turbines with both 50 and 78 m hub heights was not compared to the performance of the wind farm with a constant hub height of 78 m since the algorithm used in that study could not accommodate 22 turbines with a hub height of 78 m in the 500 m × 500 m area assigned to the farm.

In their subsequent study, Chen et al. [32] used a genetic algorithm along with PARK to find the optimal layout of a very packed wind farm with 24 wind turbines over an area of 500 m × 500 m, where there were two choices of 50 or 78 m for the hub height of each wind turbine. They also optimized a large wind farm with 18 commercial wind turbines over an area of 1200 m × 3000 m where there were two choices of 80 or 100 m for the hub height of each wind turbine. They considered three wind scenarios; first, a constant wind direction (south) with a constant wind speed of 12 m/s, second, 36 wind directions with a constant wind speed of 12 m/s, and third, 36 wind directions with 3 wind speeds of 8, 12 and 17 m/s. For the case of large wind farm with commercial wind turbines under the third wind condition, for instance, their algorithm located 15 turbines with a hub height of 100 m and 3 turbines with a hub height of 80 m to minimize cost per unit power, whereas it had located 2 turbines with a hub height of 100 m and 16 turbines with a hub height of 80 m to maximize the power output. It is worth mentioning that those layouts were not compared to layouts with similar base locations but constant hub heights of 100 m to assess the effect of using two hub heights instead of one.

According to our review, the published studies considering wind farms that include turbines with multiple hub heights did not take into account the hub height as an optimization parameter, or their optimization algorithms were based on a few possible hub heights for each turbine, mostly 2 or 3 choices, instead of doing a comprehensive search in the vertical direction, similar to the search conducted in the horizontal plane to determine the optimal base location. In addition, in those studies the hub height was not the only optimization parameter, therefore, the improvement in the objective function was not only attributed to using multiple hub heights. This prompted us to conduct an optimization analysis using a greedy search algorithm coupled with the PARK wake model by treating the hub height as the only optimization parameter to evaluate the effect of hub height alone on the overall performance of vertically staggered wind farms. The Lillgrund (Sweden) offshore wind farm with 48 Siemens SWT-2.3-365 turbines over an approximately 4000 m × 4000 m area is the test case considered here. The hub height of each turbine varies from the minimum possible hub height of 57.5 m to a maximum hub height of 100 m in 1 m increments to find the value that maximizes the annual energy production of the wind farm. The optimization was performed over 360 wind directions with a continuous range of wind speeds from the cut-in (4 m/s) to the cut-out (25 m/s) following the observed Weibull distribution.

This article consists of two major parts. In the first part (Section 2), the wind farm hub height optimization via a greedy

Table 1
Summary of the literature review.

<table>
<thead>
<tr>
<th>Farm</th>
<th>Wind</th>
<th>Method</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wang et al. (2016) [29]</td>
<td>30 large-scale wind turbines</td>
<td>PARK, BPA, Larsen and RANS models</td>
<td>Assessing the effectiveness of PARK, BPA, Larsen and RANS models for a wind farm with multiple hub heights</td>
</tr>
<tr>
<td>DuPont et al. (2016) [30]</td>
<td>A 2000 m × 2000 m area was</td>
<td>EPS - MAS algorithm coupled with the PARK model</td>
<td>Optimizing number, base location, diameter, and hub height of the wind turbines, with a cost objective function</td>
</tr>
<tr>
<td>Chen et al. (2016) [31]</td>
<td>22 wind turbines, the hub height of each wind turbine could be 50 or 78 m</td>
<td>A greedy search algorithm coupled with PARK model</td>
<td>Minimizing cost per unit power output</td>
</tr>
<tr>
<td>Chen et al. (2014) [32]</td>
<td>24 small wind turbines over an area of 500 m × 500 m, the hub height of each wind turbine could be 50 or 78 m</td>
<td>A genetic algorithm coupled with the PARK model</td>
<td>Minimizing cost per unit power output</td>
</tr>
</tbody>
</table>
heuristic search algorithm coupled with the PARK wake loss model is presented. In the second part (Section 3), four large eddy simulations are presented to validate the methodology and the PARK-based findings.

2. Wind farm hub height optimization using the PARK model

After presenting the computational methodology in Sections 2.1–2.3, the basic characteristics of wind turbines with different hub heights are studied in Section 2.4 by considering two in-line turbines aligned with the wind direction and assuming a constant wind speed $u_1 = 10 \text{ m/s}$ at a specific height $h_{ref} = 90 \text{ m}$. Then, a second case is investigated in which a wind farm with alternating rows of tall and short turbines is compared against a single-height one for the same wind direction (Section 2.5). Lastly, a complete wind farm hub height optimization is presented for a test wind farm in Section 2.6.

2.1. The PARK wake model

The PARK wake model (also known as Jensen model) assumes that the wake of a wind turbine expands linearly as wind moves downstream of the turbine. Accordingly, wind speed at the location of wind turbine $j$, which is placed at an axial distance $x_j$ downstream of wind turbine $i$ with rotor diameter of $D$, is given by:

$$u_j = u_i - u_{sw}$$

where $u_{sw}$ is the wake speed deficit and can be calculated using the PARK model.

### Table 2
Weibull parameters at different hub heights. Weibull parameters can be calculated at any given hub height using Eqs. (7)–(9).

<table>
<thead>
<tr>
<th>$h$</th>
<th>$c_w$</th>
<th>$k_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>57.5</td>
<td>9.24</td>
<td>2.39</td>
</tr>
<tr>
<td>63*</td>
<td>9.42</td>
<td>2.41</td>
</tr>
<tr>
<td>80</td>
<td>9.89</td>
<td>2.47</td>
</tr>
<tr>
<td>100</td>
<td>10.36</td>
<td>2.53</td>
</tr>
</tbody>
</table>

*a Reference height $h_{ref}$.

Fig. 1. Wind speed distribution at three different heights.

Fig. 2. Wind frequency distribution at Lillgrund (Sweden). Colors show the wind speed distribution at the reference height $h_{ref} = 63 \text{ m}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 3. Power curve of SWT-2.3-93 wind turbines used in the present study (Sections 2.6 and 3).
\[
\begin{align*}
  u &= u_c \left(1 - \frac{2\alpha}{(1 + 2kx_j/D)^2}\right),
\end{align*}
\]

where \(u_c\) is the freestream wind speed at the hub height level, \(k\) is the decay coefficient, and \(\alpha\) is the induction factor defined as \(\alpha = (1 - \sqrt{1 - C_T})/2\), in which \(C_T\) is the thrust coefficient. Using Eq. (1), the wind speed deficit at the location of wind turbine \(i\) caused by wind turbine \(j\) is

\[
\delta_{ji} = \frac{2\alpha u_c}{(1 + 2kx_j/D)^2}.
\]

Then, the total wind speed deficit at the location of wind turbine \(j\) produced by the \(n\) upstream wind turbines is calculated using the following equation,

\[
\delta_j = \sqrt{\sum_{i=1}^{n} \delta_{ji}^2}.
\]

Hence, the relative power produced by the wind turbine \(j\) is,

\[
P_{rel,j} = \left(\frac{u_c - \delta_j}{u_c}\right)^3.
\]

and the total power production of wind turbine \(j\) is calculated using,

\[
P_j = \frac{1}{2} \rho A u_c^3 P_{rel,j} C_P.
\]

where \(C_P\) is power coefficient of the wind turbine at speed \(U_c\), \(A\) is the rotor area, and \(\rho\) denotes air density.

2.2. Annual Energy Production (AEP) calculations

To calculate AEP of vertically staggered wind farms, both an extrapolation of wind speed at the desired heights and a wind direction interpolation from a coarse to a fine wind rose are needed, as described next.

2.2.1. Vertical extrapolation

The Weibull distribution, defined as:

\[
P_w(u) = \frac{kw}{cw} \left(\frac{u}{cw}\right)^{k-1} \exp\left[\left(-\frac{u}{cw}\right)^k\right] du,
\]

was used to represent the wind velocity probability density ranging from the cut-in to the cut-out wind speeds in 0.5 m/s increments, in which \(P_w\), \(u\), \(k\), and \(c\) are probability density, wind speed, shape factor, and scale factor respectively. The extrapolation method
proposed by Justus and Mikhail [35] was used to obtain the scale factor and shape factor parameters at any given height through the hub height optimization process. Using the power law wind profile, and assuming that the Weibull wind speed distribution parameters are known at a reference height \( h_{\text{ref}} \), the Weibull parameters were extrapolated at any given height \( h \) as,

\[
k_w(h) = \frac{1 - 0.0881 \ln(h_{\text{ref}}/10)}{1 - 0.0881 \ln(h/10)} k_w(h_{\text{ref}}),
\]

and

\[
c_w(h) = \left( \frac{h}{h_{\text{ref}}} \right)^n c_w(h_{\text{ref}});
\]

where \( n \) is defined as,

\[
n = \frac{0.37 - 0.0881 \ln(c_w(h_{\text{ref}}))}{1 - 0.0881 \ln(h_{\text{ref}}/10)}.
\]

Table 2 presents the reference wind data at \( h_{\text{ref}} = 63 \text{ m} \) used in this study along with the extrapolated values at sample heights of \( h = 57.5, 80, \) and \( 100 \text{ m} \). Equal Weibull parameters were used in all wind directions. The Weibull wind speed distributions associated with the three extrapolated sets of Weibull parameters given in Table 2 are plotted in Fig. 1. An increase in the height causes the maximum of the distribution to decrease and to shift to the right, meaning that the wind turbine experiences a greater wind speed.

2.2.2. Direction interpolation

The total AEP of a wind farm becomes almost insensitive to the number of wind directions at wind direction increments below 5°. Hence, wind direction increments of 1°, i.e., 360 wind directions, were used here to ensure the accuracy and the independence of the AEP from wind direction resolution. However, as is illustrated in Fig. 2, the actual wind rose available at the location of the test wind farm studied here (Lillgrund in Sweden) provided relative frequency of wind only in 12 directions [36]. Therefore, an interpolation was necessary to obtain relative frequency of wind in all 360 directions using those available in 12 directions. The interpolation was conducted as,

\[
fr_i = \frac{Fr_i}{q}
\]

where \( Fr \) is the relative frequency given in the wind rose, \( fr \) is the interpolated relative frequency, and, \( q \) and \( k \) are respectively defined as \( q = \frac{n}{m} \) and \( k = \left\lceil \frac{1}{n} \right\rceil + 1 \) in which \( n \) is the number of wind directions provided in the wind rose (\( n = 12 \)), \( m \) is the number of wind directions considered for AEP calculations (\( m = 360 \)), \( i = 1 : m \)

Fig. 5. Two in-line wind turbines aligned with the wind direction. The upstream turbine is placed at \( H_{\text{max}} \) while the hub height of the downstream turbine may vary from \( H_{\text{max}} \) to \( H_{\text{min}} \).
is the wind direction that the relative frequency $f_{ri}$ is being calculated for, and brackets $[\cdot]$ denote the integer part of the ratio $\frac{i}{q}$.

The AEP is then calculated using:

$$ AEP = \sum_{i=1}^{360} \left[ f_{ri} \times \left( \sum_{j=1}^{n_1} \left[ P_w(u_j) \sum_{k=1}^{n_1} \left( P_c(u_k) \right) \right] + \sum_{j=n_1+1}^{n_2} \left[ P_w(u_j) \frac{P_{rated}}{N_f} \right] \right) \times n_h \right] $$

(11)

where $f_{ri}$ is the interpolated relative frequency of wind in direction $i$ and is calculated via Eq. (10), $P_w(u_j)$ is the probability of having wind at speed of $u_j = (0.5 + j) du$ and is calculated by Eq. (6), $du$ is the wind speed resolution, $P_c(u_j)$ is the power obtained from the power curve of wind turbine at wind speed of $u_j$, $P_{rated}$ is the rated power of wind turbines, $n_1$ and $n_2$ are respectively defined as $n_1 = 1 + \frac{u_{cut-in}}{du}$ and $n_2 = \frac{u_{cut-out}}{du}$. $P_{rel}$ is the relative power calculated via PARK (Section 2.1), and the constant $n_h = 8760$ denotes number of hours per year. In the present paper (Sections 2.6 and 3), wind turbines were assumed to be SWT-2.3-93 manufactured by Siemens AG with a rated power of 2.3 MW, rotor diameter of 93 m, cut-in wind speed $u_{cut-in} = 4$ m/s, rated wind speed $u_{rated} = 15$ m/s, and cut-out wind speed $u_{cut-out} = 25$ m/s. The power curve of SWT-2.3-93 wind turbine is illustrated in Fig. 3.

**2.3. Optimization algorithm**

Fig. 4 illustrates the hub height optimization algorithm used in the present study. As was already mentioned, hub height $H$ is the only optimization parameter in this study and, hence, $N_f$ base locations were known in advance, where $N_f$ is the total number of wind turbines. Initially, one wind turbine with an arbitrary hub height is placed at one of the $N_f$ available locations within the wind farm area. The first wind turbine can be placed at any of the $N_f$ available locations, however, a perimetric location is preferred since it will leave the second turbine with locations that are more distant from, and consequently, less affected by the first wind turbine. Similarly, the arbitrary hub height of the first wind turbine can be anything from the minimum to the maximum possible hub heights that are set in advance; however, the minimum possible value is preferred, because after maximizing the AEP, the second objective of this optimization is to keep the farm as short as possible to reduce the capital and maintenance costs. Then, the second turbine is placed at all $(N_f - 1)$ available locations one by one, while at each location the hub height is increased from $H_{min}$ to $H_{max}$ in $dH = 1$ m increments to determine the hub height and its associated base location for which AEP of the two placed

---

**Fig. 6.** Variation of the relative power produced by the downstream turbine with its hub height for different upstream hub heights and rotor diameters. In all cases, the maximum possible hub height for the downstream wind turbine is equal to the hub height of the upstream wind turbine. The filled circle shows the hub height below which the downstream rotor starts to become unexposed to the wake produced by the upstream wind turbine, and the cross sign over each curve denotes the optimal hub height for the downstream wind turbine.
turbines is maximized. A similar procedure is applied for turbine \( n \), and \( H_{opt}(n) \) is found so that the total AEP of the \( n \) turbines which are already placed in the wind farm is maximized. This procedure continues until \( n \) reaches \( N_T \), i.e., all wind turbines with their optimal hub height are placed over \( N_T \) available locations.

2.4. Case 1: Two in-line wind turbines with different hub heights

Fig. 5 illustrates two in-line wind turbines aligned with the wind direction, where the upstream wind turbine is installed at \( H_{\text{max}} \), and the hub height of the downstream wind turbine varies between \( H_{\text{max}} \) and \( H_{\text{min}} \) in order to find the optimal hub height. The minimum hub height \( H_{\text{min}} \) is defined so that the distance between the tip of the blade and the surface never goes below 10 m. In the present section, effects of the hub height of the upstream wind turbine (\( H_u = H_{\text{max}} \)), the hub height of the downstream wind turbine (\( H_{\text{min}} \leq H_d \leq H_u = H_{\text{max}} \)), the axial distance between the wind turbines (\( d \)), the rotor diameter (\( D \)), and the surface roughness (\( z_0 \)) on the power production of the downstream wind turbine are investigated. If the hub heights of both wind turbines are equal, i.e., \( H_u = H_d = H_{\text{max}} \), the downstream wind turbine is completely immersed in the wake of the upstream wind turbine and the wake effect is at its most extreme. By lowering the hub height of the downstream wind turbine by a specific length equal to \( k \times d \), where \( k \) is the decay coefficient and \( d \) is the axial distance between the turbines, the downstream wind turbine starts to become unexposed to the upstream wake. This positively affects the power production of the downstream wind turbine. On the other hand, due to the shear effect, lowering the hub height of the downstream wind turbine causes a reduction in the speed of the wind experienced by this wind turbine, which negatively affects its power production. In many cases, a compromise between the two effects can be reached so that the maximum power production of the downstream wind turbine can be achieved at a height lower than the hub height of the upstream wind turbine \( H_{\text{max}} \). Building the downstream wind turbine at this optimal hub height (\( H_{opt} < H_{\text{max}} \)) not only increases the power production, but also slightly decreases the average height of the wind farm that may lead to a reduction of the capital and the maintenance costs. It is important to emphasize that, by lowering the hub height of the downstream wind turbine, the shear immediately shows its negative effect on the power production of that wind turbine, while the hub height should be lowered at least by \( k \times d \) to observe the positive effect of having an area that is unexposed to the upstream wake.

Fig. 6 illustrates the relative power produced by the downstream wind turbine against the hub height of this turbine for four different rotor diameters ranging from 25 to 100 m. This analysis was conducted using PARK by assuming a constant wind speed of 10 m/s at the reference height of 90 m. Wind speed at all other heights was calculated using the logarithmic boundary layer profile. Power production of the downstream wind turbine was made dimensionless using the power produced by a similar single isolated rotor installed at \( h_{\text{ref}} = 90 \) m (shown as \( P_0 \) in the legend of each figure). For the rotor diameter \( D = 100 \) m, when the upstream wind turbine is installed at \( H_{\text{max}} = 65 \) m, the optimal hub height for the downstream wind turbine is also \( H_{opt} = 65 \) m, as \( H_{\text{max}} \). In this case is too short so that the difference between \( H_{\text{max}} \) and \( H_{\text{min}} \) is smaller than \( k \times d \); hence, the downstream rotor never gets unexposed to the upstream wake and, therefore, by lowering the hub height from \( H_{\text{max}} \) to \( H_{\text{min}} \) the power produced by this wind turbine monotonically decreases under shear effect (see panel (a), left curve). If the hub height of the upstream wind turbine is \( H_{\text{max}} = 95 \) m, the downstream wind turbine can be lowered more than \( k \times d \) to get to the hub height at which a portion of the rotor becomes unexposed to the upstream wake (this hub height is shown by the filled circle in Fig. 6). The positive effect of having unexposed area is clearly revealed as the curve turns towards higher powers by further reducing the hub height; however, the increase in the power production is not enough to recover the

![Graph](image-url)
negative shear effect and the optimal hub height for the downstream wind turbine is still equal to the hub height of the upstream wind turbine. If the hub height of the upstream wind turbine is $H_{\text{max}} = 120$ m, then the hub height of the downstream wind turbine can be lowered until the positive effect of having unexposed area completely recovers the loss of power caused by the shear effect and the downstream wind turbine delivers its best performance at a height significantly lower than the maximum hub height ($H_{\text{opt}} = 57.5$ m).

Using data available for existing wind farms, the European Wind Energy Association has presented the following approximate equation for the hub height $H$ of a wind turbine with rotor diameter of $D$,

$$H = 2.7936D^{0.7663}.$$  \hspace{1cm} (12)

According to Eq. (12), the popular hub height for a wind turbine with rotor diameter of 100 m is approximately 95 m. Hence, using a hub height of $H_{\text{max}} = 120$ m for the upstream wind turbine means an approximately 26% increase in the hub height of this wind turbine; however, the hub height of the downstream wind turbine is reduced to $H_{\text{opt}} = 57.5$ m, which is about 40% decrease in the hub height of this turbine and, hence, the average hub height of the two turbines is reduced by about 14% (see Fig. 6a). More importantly, by building the two wind turbines at 57.5 and 120 m instead of installing both of them at 95 m, the total power production of the two turbines increases by approximately 3.25% (see Fig. 6a). As the rotor diameter decreases, the reduction in the average hub height of the two wind turbines becomes more significant; however, the gain in the power production reduces (see Fig. 6b-d). For instance, according to Eq. (12), the popular hub height of a wind turbine with rotor diameter of 25 m is approximately 33 m. According to our analysis (Fig. 6d), building the upstream wind turbine at 35 m and the downstream turbine at approximately 22.5 m, which is about 30% reduction in the average hub height than the case of installing both turbines at 33 m, leads to an approximately 1% improvement in the power production of the two wind turbines.

For further clarification, the two in-line turbines are assumed to be Siemens SWT-2.3-93 model with rotor diameter of 93 m, rated power of 2.3 MW and power curve given in Fig. 3. The hub height recommended by the manufacturer for this turbine model is approximately 80 m. Installing both upstream and downstream turbines at the height of 80 m gives a power production of approximately 1680 kW; however, building the upstream wind turbine at
100 m (~25% taller than the hub height recommended by the manufacturer), and the downstream wind turbine at 57.5 m (~28% shorter than the hub height recommended by the manufacturer) gives a power production of 1750 kW, which is approximately 4.2% improvement in power production while the average hub height of the two turbines is slightly reduced (see Fig. 7).

Fig. 8 demonstrates how surface roughness affects the effectiveness of using two different hub heights for the two in-line wind turbines described in Fig. 5. The negative effect of the atmospheric shear layer increases with the surface roughness, therefore, a larger fraction of the downstream rotor area needs to be unexposed to the upstream wake to make up for the losses caused by the shear effect. For instance, for a rotor diameter of 25 m, when the upstream wind turbine is installed at \( H_{\text{max}} = 40 \) m and the surface roughness is \( z_0 = 0.001-0.01 \) m (i.e., offshore condition), there is an optimal hub height lower than \( H_{\text{max}} \) for the downstream wind turbine (Fig. 8a, the right and the middle curves). Once the surface roughness is increased to about 0.1 m (i.e., onshore condition), the negative effect of the surface roughness becomes too strong so that the positive effect of having areas unexposed to the upstream wake cannot balance it and no optimal hub height lower than \( H_{\text{max}} \) is observed for the downstream wind turbine (Fig. 8a, the left curve).

By lowering the hub height of the upstream wind turbine to \( H_{\text{max}} = 35 \) m, the unexposed area of the downstream wind turbine is further restricted and only for \( z_0 = 0.001 \) m an optimal hub height lower than \( H_{\text{max}} \) is obtained for the downstream wind turbine (Fig. 8b). By further reducing the hub height of the upstream wind turbine \( H_{\text{max}} \) to values below 35 m, no optimal hub height below \( H_{\text{max}} \) can be found for the downstream wind turbine (Fig. 8c and d).

As the axial distance between the two in-line wind turbines increases, the upstream wake becomes wider at the location of the downstream wind turbine. Hence, the hub height of the downstream wind turbine needs to be lowered further to start to become unexposed to the upstream wake. This reduces the effectiveness of using multiple hub heights. For instance, for a turbine with rotor diameter of 25 m and for surface roughness of 0.001 m, when the hub height of the upstream wind turbine \( H_{\text{max}} \) is below 30 m there is no optimal hub height below \( H_{\text{max}} \) for the downstream wind turbine and it delivers its best performance when it is installed at the same height as the upstream wind turbine (Fig. 9c and d). When the hub height of the upstream wind turbine is set to be \( H_{\text{max}} = 35 \) m, an optimal hub height lower than \( H_{\text{max}} \) is found for the downstream wind turbine for axial distances

---

**Fig. 9.** Variation of the relative power produced by the downstream turbine with its hub height for different axial distances and upstream hub heights.
below 5D. When the hub height of the upstream wind turbine is increased to \( H_{\text{max}} = 40 \) m, there is always an optimal hub height lower than \( H_{\text{max}} \) for the downstream wind turbine for axial distances below 6D; however, for axial distances beyond 6D, the \( H_{\text{max}} = 40 \) m is too short to balance the increased upstream wake width and no optimal hub height below \( H_{\text{max}} \) can be found for the downstream wind turbine (Fig. 9a). So for axial distances beyond 6D, the hub height of the upstream wind turbine \( H_{\text{max}} \) must be further increased in order to have an optimal hub height below \( H_{\text{max}} \) for the downstream wind turbine.

2.5. Case 2: 20-turbine wind farm with two different hub heights

To extend applicability of the findings presented in Section 2.4, a more complicated case of a wind farm with 20 large-scale SWT-2.3-93 wind turbines, which were placed in a 5 × 4 rectangular arrangement in exact alignment with the prevailing wind in southwestern (SW) direction is considered (Fig. 10). The spacing between the wind turbines along each column is set to be identical to that of the Lillgrund wind farm in Sweden, i.e., 4.3D where \( D = 93 \) m. Two different configurations were considered. First, all 20 turbines were assumed to be at a hub height of 80 m, which is the hub height recommended by Siemens AG for this model of wind turbine (see Fig. 10a). Second, as shown in Fig. 10b, half of the wind turbines were installed at a hub height of 100 m and the other half were placed at a hub height of 100 m as the hub height optimization conducted for two in-line wind turbines with rotor diameter of 93 m indicated that using two hub heights of 57.5 and 100 m leads to approximately +4.2% more power production than using a single hub height of 80 m (see Fig. 7).

The mean power production of each row of the wind farm with multiple hub heights, defined as total power production of the entire row divided by the number of wind turbines in the row (i.e., 5), is compared against the mean power production of its corresponding row in the wind farm with single hub height. According to the PARK model (Fig. 11a), mean power productions of the first and the third rows of the wind farm with multiple hub heights are significantly larger than mean power productions of the first and the third rows of the wind farm with a single hub height. Mean power productions of the second and the fourth rows of the wind farm with multiple hub heights, however, are very close to those of the wind farm with a single hub height; the mean power production of the second row of the wind farm with multiple hub heights is slightly larger, while the mean power production of its fourth row is slightly smaller than those of the wind farm with a single hub height. According to the PARK model, the mean power production of the entire wind farm with multiple hub heights is approximately 5.4% more than that of the single hub height wind farm.

2.6. Case 3: Wind farm hub height optimization at Lillgrund

A full hub height optimization was conducted for a test wind farm with 48 SWT-2.3-93 wind turbines using the algorithm described in Fig. 4 in order to maximize Annual Energy Production (AEP). Base locations of wind turbines were identical to those of Lillgrund wind farm in Sweden, and hub height was the only optimization parameter varying from \( H_{\min} = 57.5 \) m to \( H_{\max} = 100 \) m. Although the hub height of each wind turbine could be any value from \( H_{\min} \) to \( H_{\max} \), the optimization algorithm ended up with an optimal layout in which all wind turbines were installed either at

![Fig. 10](image1.png)

(a) Test wind farm with a single hub height of 80 m, (b) test wind farm with two different hub heights of 57.5 and 100 m.

![Fig. 11](image2.png)

Time-averaged power production of each row for both single and multiple hub height configurations. Panels (a) and (b) are associated with the PARK and LES results respectively. Panel (b) will be described in Section 3.5.
\( H_{\text{min}} \) or \( H_{\text{max}} \) (see Fig. 12b) and no value between \( H_{\text{min}} \) and \( H_{\text{max}} \) was assigned to any of the wind turbines. In the optimal layout, 22 wind turbines were placed at \( H_{\text{min}} \) and 26 wind turbines were installed at \( H_{\text{max}} \). Using the PARK model, which was employed here to conduct the hub height optimization, AEP of the wind farm with single hub height of 80 m was approximately 484.236 GWh, while the AEP of the optimal wind farm was found to be 494.169 GWh, i.e., approximately 2.1% improvement. To put this into perspective, for the Lillgrund wind farm studied here and based on an electricity price of 10 cents per kWh [37], using the optimal layout with multiple hub heights of 57.5 m and 100 m instead of the original layout with a single hub height of 80 m may increase revenue by more than $1 million per year. This improvement in AEP is expected to be even more significant in real case scenarios, as the comparison between data obtained through the PARK and LES indicated that the PARK wake-loss model underestimates the improvements associated with hub height optimization (see Sections 3.5 and 3.6).

As was explained in Section 2.3, the optimization algorithm developed for this study needs to be initialized by assigning an arbitrary value to the hub height of one of the wind turbines. The optimization associated with the optimal layout illustrated in Fig. 12b was initialized by assigning a hub height of 57.5 m to the turbine located at (2843.4323 m, 3132.8249 m), which is shown via a square in Fig. 12b. The sensitivity of the optimization algorithm to this initial selection needs to be assessed to ensure that the AEP of the optimal wind farm does not drastically change by assigning the initial hub height to a different turbine. Hence, we repeated the optimization by assigning a hub height of 57.5 m to a turbine located at a different position and realized that the AEP of the optimal layout found through the new initialization is approximately equal to the previous one with only a 0.05% difference. However, the arrangement of short and tall wind turbines was approximately equal to the previous one with only a 0.05% difference. Therefore, the performance of the optimal layout only in the prevailing wind direction (southwest).

3. Validation using large-eddy simulation (LES)

After presenting the governing equations and computational details in Sections 3.1–3.4, two Large Eddy Simulations (LES) of the 20-turbine wind farm are discussed in Section 3.5, in one of which all turbines are assumed to have a hub height of 80 m, and in the other one, half of the turbines are placed at a hub height of 57.5 m and the rest of them have a hub height of 100 m. Then, in Section 3.6, two LES of the original and optimal layouts of the Lillgrund wind farm (Fig. 12a and b) are presented and discussed. It should be mentioned that an LES for a large-scale wind farm like Lillgrund is very expensive in terms of time and computational resources, as it takes approximately 60,000 CPU hours to complete one simulation only in one wind direction. Hence, it is not possible to use LES to conduct the entire optimization process or to check the performance of the optimal layout found using the PARK model in all wind directions. Therefore, we employed LES to assess the performance of the optimal layout only in the prevailing wind direction (southwest).

3.1. Governing equations

Large eddy simulations govern dynamics of large eddies by removing those with scales smaller than a filter width from the unsteady Navier-Stokes equations and modeling their effects using a subgrid scale model. The filter width is defined as \( \Delta = \sqrt[3]{\Delta x \Delta y \Delta z} \) where \( \Delta x \), \( \Delta y \), and \( \Delta z \) are cell sizes in the \( x \), \( y \) and \( z \) directions respectively. The incompressible formulations of the filtered continuity and momentum equations are as follows:

\[
\frac{\partial \overline{u}_i}{\partial x_i} = 0
\]
\[
\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u} \bar{u}}{\partial x} = -\frac{\partial \bar{p}}{\partial x} - \frac{\partial \tau^D_{ij}}{\partial x} - \frac{1}{\rho_0} \frac{\partial p_0(x, y)}{\partial x} + F_{\text{ext}} \tag{14}
\]

where the bar denotes spatially resolved components; \(i, j, \text{ and } k\) are the indices of the three spatial components \(x, y, \text{ and } z\); \(u\) is the wind speed; \(t\) is time; \(\bar{p}\) is the modified pressure defined as \(\bar{p}(x, y, z, t) = p_0(x, y, z, t) + \rho_0 g z / \rho_0 + \rho_0 \varepsilon_{ijk} / 2\); \(p_0\) and \(\rho_0\) are the static and mean pressure; \(\tau^D_{ij}\) is the traceless part of the wind stress tensor; and \(F_{\text{ext}}\) stands for the external forces applied to the wind, including those induced by the wind turbines. According to the Boussinesq eddy viscosity assumption, the traceless stress tensor \(\tau^D_{ij}\) given in Eq. (14) is defined as

\[
\tau^D_{ij} = -2\nu_t \mathbf{S}_{ij} \tag{15}
\]

where the kinematic eddy viscosity \(\nu_t\) is defined using the subgrid scale model proposed by Smagorinsky [38] as,

\[
\nu_t = (c_s \Lambda)^2 \mathbf{S} \tag{16}
\]

in which \(c_s = 0.168\) is the Smagorinsky constant, \(\mathbf{S}_{ij} = (\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i) / 2\) is the filtered strain rate tensor, and \(\mathbf{S} = \sqrt{2\mathbf{S}_{ij} \mathbf{S}_{ij}}\) is the norm of the filtered strain rate tensor. The external force \(F_{\text{ext}}\) term in Eq. (14) includes the Coriolis force, the buoyancy force, and the force exerted by turbine blades that is calculated using the actuator line model presented in Section 3.2. Accordingly, the external force \(F_{\text{ext}}\) can be expressed as,

\[
F_{\text{ext}} = \frac{1}{\rho_0} F_i + g \left( \frac{\theta - \theta_0}{\theta_0} \right) \delta_{ij} - \epsilon_{ijk} f \bar{u}_k \tag{17}
\]

where \(F_i\) is the force generated by the actuator line model, \(\epsilon_{ijk}\) is the alternating unit tensor, \(g\) stands for the gravitational acceleration, \(\theta\) is the potential temperature, \(\theta_0 = 300\) K is the reference temperature, \(\delta_{ij}\) is the Kronecker delta, and \(f\) is the Coriolis parameter defined as \(f = 2\Omega \sin \phi\) in which \(\Omega\) is the Earth rotational speed \((\sim 2.95 \times 10^{-5}\) rad/s), and \(\phi\) is the site latitude. The following potential temperature equation needs to be solved coupled with Eqs. (13) and (14) to obtain the potential temperature needed to calculate the buoyancy term in Eq. (17),

\[
\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial (\bar{u} \bar{\theta})}{\partial x_j} = \frac{\partial q_j}{\partial x_j} \tag{18}
\]

where \(q_j\) represents the temperature flux defined as

\[
q_j = -\nu_t \frac{\partial \bar{\theta}}{Pr_t \partial x_j} \tag{19}
\]

and \(Pr_t\) is the subgrid turbulent Prandtl number defined as [39],

![Fig. 13. Time variation of the mean power production of the 20-turbine wind farm over 15 min of operation as simulated by SOWFA.](image1)

![Fig. 14. Wind speed distribution on a vertical plane along with the most southern column; (a) single hub height (80 m), (b) multiple hub heights (57.5 and 100 m).](image2)
\[
Prt = \frac{1}{1 + 2\lambda^2}
\]

in which,
\[
l = \begin{cases} 
\min \left(7.6 \frac{\lambda}{\Delta} \left(s^2\right), \Delta \right) & \text{if } s > 0 \\
\Delta & \text{if } s \leq 0
\end{cases}
\]

and
\[
s = \frac{g}{\partial \partial_2}.
\]

Usually \(l = \Delta\), and hence, \(Prt = \frac{1}{1}\).

### 3.2. Wind turbine modeling

The actuator line modeling, proposed by Sørensen and Shen [40], was employed along with large eddy simulations to model the wind turbines. In this model, the turbine blades are represented by three rotating lines that are discretized into 40 blade elements with centers located at \((x_n, y_n, z_n)\). Using airfoil lookup tables, the aerodynamic forces are calculated for each blade element \(f_a(x_n, y_n, z_n, t)\). Summation of the aerodynamic forces of blade elements corrected via a regularization kernel yields the body force exerted by the blades onto the flow field,

\[
F_i = \sum_{n=1}^{40} f_a(x_n, y_n, z_n, t) \frac{r_n}{\pi r_n^2} \exp \left[ -\left( \frac{r_n}{\ell} \right)^2 \right],
\]

where \(f_a(x_n, y_n, z_n, t)\) is the actuator element force, \(F_i\) is the force field projected as a body force onto CFD grid, \(r_n\) is the distance between CFD cell center and the blade element, and \(\ell\) is used to control the Gaussian width so that it spans from the leading edge to the trailing edge of the blade elements. In the present work, \(\ell\) was set to be \(\ell/4.3\), where \(\ell\) indicates the chord length of the blade elements, so at both trailing and leading edges (i.e., \(r_n = \ell/2\)) the exponential term was reduced to approximately 1% of its maximum [41]. The power calculations are based on the aerodynamic torque that is exerted on the blades. Multiplying the aerodynamic torque by the rotational speed of the rotor yields the power output.

### 3.3. Numerical solution

The governing equations described in Sections 3.1 and 3.2 were solved using SOWFA (Simulator for Wind Farm Applications), which is a CFD solver based on the OpenFOAM toolbox developed by the United States National Renewable Energy Laboratory [42]. The equations were discretized using a finite volume formulation. A second order central differencing scheme was used to conduct the spatial discretization since the large eddy simulations are too sensitive to the false numerical diffusion, and the central differencing scheme significantly reduces the numerical diffusion in comparison with the upwind scheme. The Pressure Implicit with Split Operator (PISO) [43] algorithm was used for the pressure–velocity coupling. Comparing to other pressure–velocity coupling methods, such as SIMPLE [44] and SIMPLEC (SIMPLE-Consistent) [45], the PISO algorithm requires more CPU time per solver iteration; however, it significantly helps to maintain a stable calculation with larger time steps, and hence, decreases number of iterations.

### 3.4. Simulation setup

The simulations were conducted as described in Archer et al. [15], Vasel-Be-Hagh and Archer [17], and Xie et al. [34]. The computational area was a rectangular cuboid with width, depth and height of 4000 m, 4000 m, and 1000 m respectively. At first, a coarse mesh with approximately 5,900,000 hexahedral cells was created using the blockMesh utility supplied with OpenFOAM. This mesh was then refined locally around the wind turbines leading to approximately 22,000,000 cells. The simulations were conducted in two steps. First, a precursor simulation was carried out to develop the flow field through the computational domain without taking into account the wind turbines for 12,000 s real time, and Fig. 15. Wind speed distribution on horizontal planes. Panel (a) is associated with the single hub height wind farm described in Fig. 10a and both panels (b) and (c) are associated with the multiple hub height wind farm described in Fig. 10b. Panel (b) shows the wind speed distribution at the hub height of short turbines (57.5 m) and panel (c) illustrates the wind speed distribution at the hub height of tall turbines (100 m).
then, a wind plant simulation was performed for 2000 s by adding the wind turbines into the developed flow field. Conducting the wind plant simulation over the above-described computational domain requires approximately 60,000 h CPU time using a 2.4 GHz processor of a high-performance computing cluster. The present study was conducted using 192 parallel processors.

A horizontally-averaged wind speed of 9 m/s was specified at the height of 90 m in the south-west direction, i.e., 225° from the north using meteorological convention. The wind speed boundary condition was set as slip at the top and bottom boundaries, outlet at the north and east boundaries, and inlet with time-varying mapped fixed values at the west and south boundaries. The buoyant pressure boundary condition was used at all six boundaries of the domain to define the pressure boundary field. The temperature boundary condition was set as fixed gradient with a uniform value of 0.003 K m/s at the top boundary, zero gradient at the bottom, east and north boundaries, and time-varying mapped fixed values at the south and west boundaries.

3.5. LES of flow over the 20-turbine wind farm with two different hub heights

To validate the findings presented in Sections 2.4 and 2.5, two large eddy simulations were conducted to calculate power production of the 20-turbine wind farm previously described in Fig. 10. Unlike results obtained through the PARK model, LES shows a significant improvement in power production of the second row of the multiple hub height wind farm (Fig. 11b) meaning that PARK model underestimates the effectiveness of using multiple hub heights and using multiple hub heights may lead to even more significant gains in power production in reality compared to what is predicted by the PARK model. This underestimation can be attributed to the simplifying assumptions underlying the PARK model. As was explained in Section 2.1, the PARK model is a simple analytical wake loss model which simply assumes that the wake of a wind turbine expands linearly without any meandering and with an uniform wind speed throughout the wake at each distance downwind. PARK does not take into account turbulence intensity, turbulence length scales, or atmospheric stability conditions. In addition, the wake overlapping is simply performed by taking the square root of the sum of the squared wind speed deficits induced by the wind turbines (see Eq. (3)), which is not perfectly consistent with reality. Despite all of these simplifications, the PARK model has been proven to be one of the most effective and accurate analytical wake models, but not as accurate as sophisticated LES, which predict the wake properties by solving the Navier-Stokes equations. The accuracy of large eddy simulations is at the expense of computing time and they are in the order of 10⁶ times more time-consuming than PARK, and hence, they cannot be used for optimization purposes.

The farm-averaged power production, defined as the average of the instantaneous power produced by all the wind turbines, is almost always greater in the multiple hub height configuration than in the single hub height configuration (Fig. 13), approximately 889.52 kW and 811.98 kW, respectively. Thus, according to LES, using the multiple hub height configuration increases the power production of this wind farm by approximately 9.5% in the studied wind direction, whereas the gain was only approximately 5.4% with PARK.

The mean wind speed distribution along the most southern column of the wind farm (C1 in Fig. 10a) over a two-dimensional vertical plane (Fig. 14) indicates that, if turbines are vertically staggered, they are less affected by the wake of each other provided that the vertical spacing between them is sufficiently large. In the column with wind turbines with hub heights of 57.5 and 100 m (Fig. 14b), the lower half of the second wind turbine and the upper half of the third wind turbine are exposed to higher wind speeds; while in the column with wind turbines with single hub height of 80 m (Fig. 14a), the second and the third turbines are immersed in the upstream wake. To provide further clarification, the mean wind speed distribution is presented on horizontal planes at three hub height levels in Fig. 15. The length of the upstream wake at the hub level is much shorter in the wind farm with multiple hub heights than the wind farm with a single hub height, which leads to a higher power production.

3.6. LES of the optimal layout in the prevailing wind direction at Lillgrund

To validate the optimal layout found for Lillgrund wind farm using the PARK model (Fig. 12b), two large eddy simulations were conducted in the prevailing wind direction (southwest). In the first simulation, all wind turbines were assumed to be installed at 80 m, i.e., the original layout, and in the second simulation, 22 wind
turbines were placed at 57.5 m and 26 wind turbines were installed at 100 m, i.e., the optimal layout presented in Fig. 12b. Based on the time history of the farm-averaged power production of these two layouts (Fig. 16), the optimal layout with multiple hub heights produces more power than the original single hub height layout almost all the time. Time-averaged value of the farm-averaged power productions of the optimal and original layouts are approximately 829.1 kW and 780.1 kW, respectively, i.e., approximately 6.3% improvement. This improvement in the power production in the prevailing wind direction was predicted to be only approximately 3% using the PARK model, demonstrating that the PARK model underestimates the gain in power due to the hub height optimization and the AEP improvement for the optimal layout will be even more than 2% in reality. Looking at the wind speed distribution through the wind farm for both original and optimal layouts over two dimensional planes at the hub height level (Fig. 17), it is clear that wind turbines located downwind of a taller or a shorter wind turbine are subjected to weaker upstream wakes.

4. Conclusions

Turbine hub height was taken into account as the only optimization parameter to minimize adverse wake effects in wind farms. To ensure the effectiveness of vertically staggering wind turbines, hub height was assumed to be the only optimization variable, while number of turbines, rotor diameter, and base location of wind turbines were set to constant values in advance and remained unchanged during the optimization. The work was carried out in two parts.

In the first part, the effect of using two different hub heights was studied using the PARK wake model and considering two in-line wind turbines. It was found that, owing to shear effect, lowering the hub height of the downstream wind turbine initially causes a reduction in the power production; however, once the difference between the hub height of the upstream and downstream wind turbines goes beyond a certain value, the power production increases with further reduction in the hub height of the downstream wind turbine, so that it may go even beyond the power produced by two turbines with equal hub heights. It was also found that the idea of using multiple hub heights is more applicable for wind farms that are more packed (with axial spacing below \( \frac{C_2}{D} \)) and are located offshore (with surface roughness below \( \sim 1 \text{ mm} \)). Then, a full hub height optimization was conducted for a test wind farm with 48 large-scale wind turbines. The base locations of the wind turbines of this test wind farm were identical to those in the Lillgrund wind farm in Sweden. The optimization was conducted in 360 wind directions. While the hub height of each wind turbine could be any value between \( H_{\text{min}} \) and \( H_{\text{max}} \), it was found that the wind farm performs best when turbines are installed at either \( H_{\text{max}} \) or \( H_{\text{min}} \). The distribution of turbines with \( H_{\text{max}} \) and \( H_{\text{min}} \) depends on the initialization of the optimization, but the AEP of the optimal layout was approximately independent of the initialization and, for the test wind farm studied here, was found to be approximately 2% more than the AEP of the wind farm with a single hub height.

In the second part, large eddy simulations were conducted to validate findings of the first part. First, two large eddy simulations were conducted for two wind farms with 20 large-scale wind turbines to investigate the effectiveness of our findings using two in-line wind turbines for a realistic wind farm. In the first wind farm all turbines were installed at the same height, while in the second wind farm half of the wind turbines were installed at \( H_{\text{max}} \) and the other half were placed at \( H_{\text{min}} \). Using two different hub heights caused the average power production to increase by approximately 9.5%. Finally, large eddy simulations were conducted to assess the

Fig. 17. (a) Wind speed distribution through the original layout at the hub height of wind turbines (\( H = 80 \text{ m} \)). (b) Wind speed distribution through the optimal layout at the hub height of short wind turbines (\( H = 57.5 \text{ m} \)). (c) Wind speed distribution through the optimal layout at the hub height of tall wind turbines (\( H = 100 \text{ m} \)).
performance of the optimal layout found using the PARK model. The farm-averaged power production of both original and optimal layouts were calculated in the prevailing wind direction and the optimal layout was found to produce approximately 6.3% more power compared to the original layout in the prevailing wind direction. By comparison, the PARK model predicted the gain to be approximately 3%, which demonstrates that PARK under-predicts the power production improvements caused by the wind farm hub height optimization by up to a factor of two.

Acknowledgments

All simulations were conducted on the Mills and the Farber high-performance computing clusters of the University of Delaware. This work is made possible by the University of Delaware and the First State Marine Wind LLC.

References